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Determination of the multi-scattered solar radiation from a leaf canopy for use in climate models

Robert E. Dickinson *

Earth and Atmospheric Sciences, Georgia Institute of Technology, 311 Ferst Drive, Atlanta, GA 30332-0340, United States

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Abstract

The terrestrial component of climate models requires computationally efficient algorithms for determining the multi-scattered radiation contributing to its heating from solar radiation. Much of the vegetated land cover has strong 3D controls on its radiation. The scattering from a 3D object of isotropic scatters is formulated abstractly and an approach to solution is described in the context of a spherical object. A Laplace integral representation of the 3D integral equation for radiative transfer is discretized. Such discretization provides the solution in terms of solutions to 3D Helmholtz equations, a single such equation in the lowest order approach. A Green's function approximate solution along the paths of entering and exiting radiation is integrated over such radiation for the paths assumed to coincide except for direction. The resulting approximate description of multi-scattered radiation corresponds to replacing the 3D scattering paths with a 1D path with attenuation amplified by a diffusivity factor. This description combines with previously derived analytic solutions for single scattered radiation to provide an efficient representation of the bidirectional scattering from the 3D object, intended for use in climate models and remote sensing.

Keywords: Remote sensing; Climate modeling; Scattering; Land surface model; Bidirectional reflectance; Land albedo; Land radiation

1. Introduction

The terrestrial component of climate models addresses the physics of energy and water exchanges at the land surface. In particular, it calculates the balance between net radiative heating and turbulent fluxes of latent and sensible heat [1]. The radiative forcing is in part determined by transmission through the atmosphere of solar radiation incident at the top of the atmosphere. It also involves exchanges of thermal radiation and the determination of how much solar radiation is reflected back toward the atmosphere. It is the latter topic that is addressed here. Similar issues are encountered in treating the effects of clouds in the atmosphere [2].

Climate models do direct computation on time scales from tens of minutes to centuries. Furthermore, they do these computations on a global mesh with as high resolution as feasible and treat as many physical details

* Tel.: +1 404 385 1509; fax: +1 404 385 1510.
E-mail address: robted@eas.gatech.edu

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as possible. With the advent of global quantitative derivation of terrestrial properties by remote sensing, it is in principle possible to calculate the global terrestrial system with a resolution as fine as one kilometer. Thus only computationally very simple algorithms, such as evaluation of a small number of exponentials, may be practical to use for describing the terrestrial system in climate models.

Solar radiation after passing through the atmosphere arrives either in the form of direct beam or diffuse, that is after molecular (Rayleigh) or particulate scattering from cloud or aerosol droplets. This radiation passes through vegetation canopies to reach the surface and is either absorbed by the canopy or surface or is reflected back to the atmosphere, e.g. [3]. The treatment of the radiation passing through vegetation canopies is complicated by the presence of multiple levels of organization, from that of the chloroplast cells to that of the arrangement of individual plants within the landscape. Thus, a comprehensive treatment of all the radiative details within a 1 km^2 plot of land could easily require by itself enormous computation. How can we reduce the computation of terrestrial radiation to something very simple but still adequately realistic?

Radiation interacting at multiple scales of organization can be addressed with an adding principle. Each level is summarized by its "input–output". That is, since the incident radiation is of external origin, it arrives at the outside of an object, enters in and some fraction again exits in various directions. The description of this exiting radiation for a given input is referred to as "optical properties". The optical properties of a plant cell can be used to construct the optical properties of a leaf and the latter can be used to construct optical properties at higher levels of organization up to that of the canopy.

The scattering objects that are considered in treating radiation within a canopy are generally not opaque. Radiation both reflects from a leaf surface and is transmitted diffusely through it. However, this scattering at leaf level is commonly asymmetric, i.e., the fraction of incident light reflected from the surface of a thick leaf differs from that transmitted. Scattering from leaves is further complicated by the geometry of leaf orientation which is commonly characterized by a statistical distribution.

These leaf optical properties have been successfully included for homogeneous canopies (i.e., no higher level of organization) at the computational level useful in climate models. For this purpose, the scattered radiation has been conceptualized as consisting of discrete streams, e.g. 2-stream [3–5], or 4-stream [4,6,7] or alternatively, been represented by polynomials in the cosine of their angle made by the direction of the radiative flux relative to same reference direction, e.g. the vertical. These approaches are low order numerical discretizations of the continuous directionality of scattered radiation. They describe the scattering from terrestrial vegetation with the computational simplicity needed for a climate model and with acceptable discretization error [6]. They are not able to explicitly address any of the additional complexity of the geometric organization of systems at lower levels in the hierarchy, in particular that of individual trees or bushes.

For some climatically important questions, differences in the geometric properties of the trees arguably exert more control on the absorption and reflection of radiation than do the properties of the individual leaves. The effects of the woody components (i.e., trunks and branches) can also be substantial. Consequently, the terrestrial remote sensing community has recognized the importance of including 3D geometric effects. This complexity has been included through computationally intensive numerical modeling that represents the radiative properties of a canopy in terms of lookup tables for assumed canopy shapes but with the projected leaf area index (LAI) treated as a variable [7,8]. The most flexible and realistic treatment of canopy radiation for a particular set of parameters is that of "Monte-Carlo" since any and all geometrical configurations can be included in the context of statistical choices. Several such codes and other more complex 3D treatments have been compared in "RAMI" [9] One possible approach to the simplification needed for a climate model is to use such relatively exact (and computationally intensive) computation to select parameters of a simple model. This approach has been studied [10] for fitting parameters of a 2-stream plane parallel model.

Satellites observe radiation reflected in their direction whereas climate models need total solar energy reflected upward in all directions. This distinction has led to the optical properties of vegetation canopies being represented somewhat differently in climate models than they are in remote sensing. However, the current trend in meteorological modeling and remote sensing is to generate the remotely sensed signal in the meteorological model and use the difference from that observed to correct the model through data assimilation procedures. Because of the likely continuation of global terrestrial climate records through instruments on meteorological satellites, data assimilation of terrestrial information should be further developed beyond its current framework for soil moisture [11]. For this purpose, the computation of radiation in climate models needs to be formulated to reproduce the directional information seen by a satellite. Thus, the question addressed here is: how can we: (a) compute realistic 3D effects of radiation efficiently enough to be useful for a climate model; and (b) so that it would also be useful for interpretation of remotely sensed radiation? Detailed analytic solutions are presented in [12] in the single-scattering limit. The primary concern of this paper is the development of an analytic approach for the treatment of the additional multi-scattered radiation.

This paper models leaves as homogeneously distributed centers of isotropic scattering. The additional complexities of leaf orientation, scattering asymmetries, or woody components are not addressed. The paper assumes a spherically shaped cloud of scatters, visualized to be a "spherical bush" and its component leaves. "Homogeneous" here refers to a random distribution of leaf locations within the prescribed 3D geometric object. For 1D models, it refers to such a distribution between two planes.

The mathematical theories of radiative scattering have already extensively developed especially in the context of astrophysics and neutron diffusion where the system-properties differ significantly from those considered here. In particular, the astrophysical scattering systems are commonly very deep and appropriately approximated as semi-infinite, whereas the study of neutron scattering has been focused on the issue of "criticality" where as much or more energy is scattered as is incident. Plant canopies on the other hand are often optically relatively thin and at most wavelengths their leaves absorb a substantial fraction of the radiation they attenuate. Thus a different analytic approach for determination of their scattered radiation is needed and contributed to here.

A previous approach, most extensively popularized by Chandrasekehar [13], and Sobelev [14], leads to simple but nonlinear integral equations whose solution provides the exiting radiation. The approach developed here is to directly use the linear integral equation that describes the scattering within the interior of an object.

2. Formal operator description

The problem addressed is that of unit radiation entering a 3D object with a discrete or continuous distribution of scattering centers. What fraction of radiation again exits as a function of its exiting direction? The interactions of the radiation with the leaves are determined in the limit of geometric optics. The direct beam transmission and the single scattered radiation are derived from simple integrations described in [12] and briefly summarized in the next sections for their use with the multi-scattered radiation.

The radiation that is attenuated per unit depth at some location within the object is denoted $A(x, yz)$ or simply $\mathbf{A}$. It corresponds to the local flux intensity multiplied by an optical depth parameter τ, i.e., optical depth per unit distance, and provides a source term for scattered radiation.

The "formal" operator description of this section is provided to help clarify the structure of multiple scattering before in the next sections addressing concrete formulations for determining such. The equation assumed to apply for $\mathbf{A}$ is that

$$\mathbf{A} = \mathbf{A}_0 + \omega\mathbf{B}\mathbf{A}, \tag{1}$$

where $\mathbf{B}$ is referred to as the "bush operator", and $\mathbf{A}_0$ is the attenuation of the direct solar beam. The second term on the right side is the contribution of radiation that has been attenuated more than once. If $\mathbf{A}$ is viewed as a matrix, then $\mathbf{B}$ is another matrix that describes how the elements of $\mathbf{A}$ are coupled together by scattering. The term $\mathbf{A}_0$ is constructed by solving some version of the "Lambert–Bouguet–Beer Law" of exponential decay for all paths along the direction of the entering solar radiation. This "Law" states that the attenuation per unit distance of the radiation in a given direction for an infinitesimal increment of path is simply the "optical depth"τ (c.f., [15, p. 52]). Once we have determined the source term $\mathbf{A}$, the outgoing flux $\mathbf{\Omega}$ (in some direction relative to that of the incident radiation) is constructed from the path of outward scattered photons, i.e.,

$$\mathbf{\Omega} = \mathbf{E}\mathbf{A}_0. \tag{2}$$

The term $\mathbf{E}$ varies with location in the object, and is the exponential decay from that location to a point on the boundary in the selected direction of exiting radiation. Except for referring to a different direction, it is the same term as used to determine $\mathbf{A}_0$.

The resolvent bush operator $\mathbf{\Gamma}$ is simply the expression obtained by solving Eq. (1) in the form:

$$\mathbf{A} = \mathbf{A}_0 + \omega\mathbf{\Gamma}\mathbf{A}_0. \tag{3}$$

If we assume that the operators can be manipulated as scalars,

$$\mathbf{\Gamma} = (1 - \omega\mathbf{B})^{-1}\mathbf{B}. \tag{4}$$

This formal description of the solution for $\mathbf{\Gamma}$ is more straightforward to flesh out with details when $\mathbf{B}$ is a discrete matrix with eigen-vectors. That is, we would like to be able to expand the identity matrix $\mathbf{U}$ whose elements are δ_{ij} as

$$\mathbf{U} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{\Psi}, \tag{5}$$

where the matrix $\mathbf{\Psi}$ consists of column vectors that are eigen-vectors of $\mathbf{B}$. That is, it satisfies:

$$\mathbf{B}\mathbf{\Psi} = \mathbf{\Lambda}\mathbf{\Psi}, \tag{6}$$

where $\mathbf{\Lambda}$ is the diagonal matrix $\delta_{ij}\lambda_i$ and the λ_i are the eigen-values of $\mathbf{B}$. If this diagonalization can be done, we can manipulate Eq. (6) to $\mathbf{B} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^{\mathrm{T}}$ interpreted as a projection into the eigen-space of $\mathbf{B}$, and so determine $\mathbf{\Gamma}$ as

$$\mathbf{\Gamma} = \mathbf{\Psi}(1 - \omega\mathbf{\Lambda})^{-1}\mathbf{\Lambda}\mathbf{\Psi}^{\mathrm{T}}. \tag{7}$$

Depending on whether it is described in terms of discrete elements or as a continuum, a given location is connected to all other locations in a given direction by a sum over all these locations or by an integral over paths in a given direction. The solution represented by Eq. (3) can include in addition any term that satisfies $(\mathbf{U} + \omega\mathbf{\Gamma})\mathbf{A} = 0$. Such "homogeneous eigensolutions" may be needed to satisfy physical requirements of the solution.

Besides simply describing the scattering exchange between different locations in a canopy, the $\mathbf{B}$ introduced in Eq. (1) has at least two other interpretations described here in their discrete version: (a) it acts as a correlation matrix; that is if the canopy is exposed to white noise forcing, $\mathbf{B}$ will simply describe how radiation at one point is correlated with radiation at other locations inside the bush. It also is a smoothing matrix. If radiation is applied at one location in the bush, $\mathbf{B}$ acts to smear out the radiation over a distance $d = 1/\tau$.

3. Continuum unit sphere model

The abstract expressions of the previous section with appropriate details apply to either a discrete or continuum representation of multiple scattering from a canopy. Although the most realistic viewpoint is that of discrete leaves, such can only be addressed by Monte-Carlo sampling. Other coarser grained discrete formulations for canopy scattering generally should be derivable from a continuum viewpoint, so to be specific, we adopt such a viewpoint that is basically probabilistic, i.e., leaves are included by spatial statistical distributions. As the intent of this study is to isolate the aspects of 3D geometry contributing to canopy scattering, we make the simplest assumptions about other parameters, in particular that the leaves are homogeneous in space with isotropic orientations, and furthermore, that they scatter isotropically; that is leaves have a scattering phase function that is a constant, independent of the angle between the incident and reflected photons. These assumptions allow us to address concrete versions of the abstract problem described in the previous section, to establish the geometric controls on the scattering of solar radiation from a distribution of idealized leaves.

The simplest such geometry to study is that of a sphere. For this geometry, some aspects of the single scattered reflected radiation can be characterized by analytic integrations [12] and these were shown by numerical computation to be useful for constructing a complete scattering phase function for single scattering. Multiple scattering was also included, but by use of an approximate expression that does not provide adequate accuracy except for a thin canopy. The radiation that was attenuated and not lost to single scattering was used as a uniform source term for the multiple scattering. An objective of the present paper is to establish how the non-uniform volume spatial distribution of the attenuation of the incident solar radiation creates a non-uniform source term for the multiple scattering. The details of this source term involves 3D integrals which are difficult to describe and can only be evaluated accurately by numerical computation. However, the abstract analysis of the previous section and the study of [16] suggest that a simple analytic representation of the multiple-scattering source term may provide an expression with much better accuracy than the uniform source treatment.

For a continuum, the bush attenuation variable $A(\mathbf{r})$ satisfies an integral equation:

$$A(\mathbf{r}) = A_0(\mathbf{r}) + \omega \int \mathrm{d}\mathbf{r}' A(\mathbf{r}')\mathbf{I}(\mathbf{r},\mathbf{r}'), \tag{8}$$

where A is as before, the radiation attenuated per unit volume, $\mathbf{r}$ is a 3D Cartesian vector (x, y, z), so $|\mathbf{r} - \mathbf{r}'|$ will denote the scalar distance between the points $\mathbf{r}$ and $\mathbf{r}'$. The kernel $\mathbf{I}$, which with integral is an example of the abstract bush operator $\mathbf{B}$ introduced earlier, describes the modification of radiation emitted from a point source at $\mathbf{r}'$ and attenuated by exponential decay until it arrives at $\mathbf{r}$. The radiation spreads from the point source with its intensity proportional to the inverse square of the radial distance from its origin, i.e., the same as radiation leaving the sun treated as a point source:

$$\mathbf{I} = \tau \exp(- \mid \mathbf{r} - \mathbf{r}'|\tau)/(4\pi \mid \mathbf{r} - \mathbf{r}'|^2). \tag{9}$$

The radial spreading term expresses the constancy of the energy flux per unit solid angle in the absence of the exponential attenuation.

The 3D kernel $\mathbf{I}$ as written is not analytically integrable. However, it can be reduced to a sum of terms that are as follows: it is written as a Laplace integral [17] that has the effect of replacing the $|\mathbf{r} - \mathbf{r}'|^2$ in its denominator with a $|\mathbf{r} - \mathbf{r}'|$,

$$\mathbf{I} = \tau \int_\tau^\infty \mathrm{d}p \exp(-p|\mathbf{r} - \mathbf{r}'|)/(4\pi|\mathbf{r} - \mathbf{r}'|). \tag{10}$$

Although at first blush, seeming to simply complicate Eq. (9), the transform representation Eq. (10) has a beneficial effect for development of an analytic approach to solution. This transform kernel is recognized to be the solution $P(\mathbf{r}, p)$ for a 3D Helmholtz Green's function equation

$$[\Delta - p^2]P = -\delta(\mathbf{r} - \mathbf{r}'), \tag{11}$$

where Δ is a 3D Laplacian. Analytic solutions for Eq. (11) are well known for various coordinate systems [18]. Although spherical coordinates might appear to be the most obvious choice, a local system with a Cartesian metric along the path of entering radiation was shown in [12] to be analytically integrable. The dependence of these solutions on p is addressed by introducing a numerical integration of Eq. (10). The integration in Eq. (10) can also be written in terms of another transform variable, $q = 1/(\tau p)$ which puts it in the form most familiar for plane parallel systems [13], with q interpreted as the cosine of the angle of the direction of the radiation. It has been established over the history of radiative transfer research with such systems that a few integration points, even one (e.g. the classical 2-stream and Eddington approximations [19]), provides useful accuracy for integration of expressions corresponding to Eq. (10).

Uniform intervals in q appear to provide relatively accurate collocation, corresponding to the 2-stream and 4-stream models described in [19] and assessed in [6]. For this choice, and for a single integration point, we assume in Eq. (11) that $q = 0.5$, and $\mathrm{d}p = -q^{-2}\mathrm{d}q = 4$. The resulting approximation is exactly the same as the commonly used "diffusion approximation" and corresponds to the physical reasoning that the 3D radiation can be addressed in a single dimension by allowing for an average cosine projection factor for the radiation. The factor $p = 1/q = 2$ provides an exact integration over the directionality of the radiation only in the limit of small paths. For the plane parallel systems, the exact integration can be expressed as an exponential integral and various approximations used to put this in a simpler form (e.g. [10]). Physically, radiation over longer paths becomes more concentrated in the forward direction such that the effective diffusivity factor $\rightarrow 1$ as $\tau \rightarrow \infty$. However, for open vegetation canopies, we are dealing with relatively small vertical optical paths and a 3D structure that acts to greatly weaken the dependence of path length on the angle of the radiation relative to a normal to the surface. Hence, a diffusivity factor of 2 is expected to provide substantially greater accuracy for integrations over an open 3D vegetation canopy than that for plane parallel canopy systems. The diffusivity factor of 2 corresponds to the two-stream approximation now commonly used in climate models to address details of canopy radiation as a plane parallel system.

The basic complexity of Eq. (8) originates from its inclusion of contributions from all locations within the unit sphere. In the approximation now to be described, based on the above theoretical arguments, these locations are connected to a path along that of the incident radiation. Integration over the sphere can be reduced to integration over a local coordinate as measured from point of radiation entry h as indicated in Fig. 1, cf. [12] for further discussion. The second coordinate μ enters only as a description of the location of the sphere's boundary (i.e., $\mu =$ the cosine of the angle the entering radiation makes with a direction normal to the surface of the sphere), and the volume integration is done by first integrating in h, then over μ where $2\mu\,\mathrm{d}\mu$ is the sphere's relative area normal to the path of radiation.

The attenuated radiation $A_0(h,\mu)$ in these coordinates is simply

$$A_0(h,\mu) = \tau\exp(-h\tau). \tag{12}$$

Multiple scattering within the geometry used and leading to downward scattering is illustrated in Fig. 1. The parameter μ enters only in describing the entry point and end point of the local path. We proceed as follows. First, for the remainder of this section, as a relatively simple and informative result, we develop the second order scattering contribution to A. The next section shows that with some more mathematical development, essentially the same approach provides scattering to all orders, i.e., providing a concrete example of the resolvent bush operator introduced by Eq. (4).

The second order scattering contribution to A, denoted A_{2s}, and corresponding to substituting $\mathbf{A}_0$ in place of $\mathbf{A}$ in the integral in Eq. (8) is now obtained as

$$A_{2s}(h,\mu) = \int_0^{2\mu} \mathrm{d}h' B(h,h')A_0(h'). \tag{13}$$

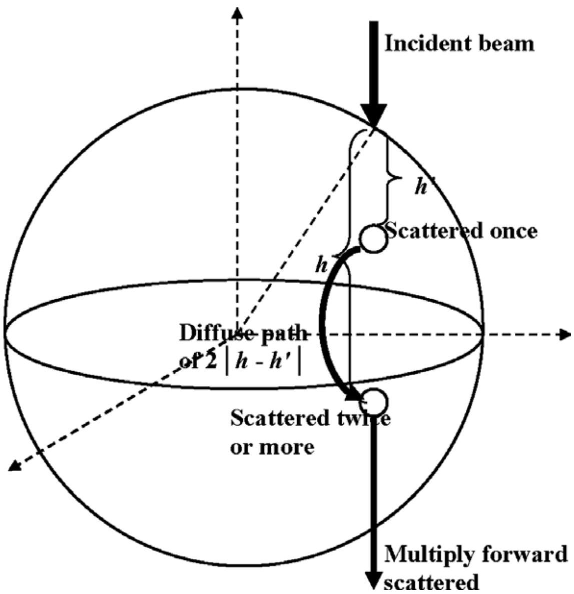

Fig. 1. Sketches the geometry of the integration paths over the sphere for forward scattering. The sphere's axis is normal to the direction of the incident radiation. An individual beam enters the sphere at a point where surface normal makes an angle relative to the vertical axis whose cosine is μ. The column the beam passes through is of length 2μ. Along this path, the incident beam travels a distance h' and then is scattered to some other location in the sphere. The integration path to the other location is approximated by an integration over the initial column to point h but with the path amplified by a diffusivity factor of 2. This approximation corresponds to a 1 point integration over a Laplace transform representation of the exact solution. More integration points would produce more terms requiring different diffusivity factors. From point h, photons are scattered forward to exit the sphere. The total forward scattering is then provided by integrating such columns over the scaling parameter μ.

The "spherical bush kernel" derived with the "diffusivity approximation" of the theory just described is simply

$$B(h,h') = \tau \exp(-2\tau|h-h'|), \tag{14}$$

providing a 1D approximation to **I** of Eq. (9). The first derivative of B jumps from 2 for h less than h' to -2 for h greater than h', i.e., a jump of 4 reflecting the definition of a Green's function multiplied by the earlier mentioned factor of 4.

With substitution of Eq. (12) and (14), Eq. (13) integrates to

$$A_{2s}(h,\mu) = B^*(h,\mu)A_0(h), \tag{15}$$

where

$$B^*(h,\mu) = 4/3 - \exp(-h\tau) - 1/3\exp[-3(2\mu-h)\tau]. \tag{16}$$

In the limit of small τ, the right hand side of Eq. (16) reduces to 2μ or simply the length of the path of integration.

In addition to the "diffusivity" approximation, we also have assumed in obtaining Eq. (14) that in Eq. (11) the variation of P in directions normal to the path can be neglected, an assumption that is least in error for scattering paths in the forward and backward directions relative to the initial path. Fortunately, these are the directions whose integrals are needed to estimate the multi-scattering contribution to the scattering phase function using the approach of [12], where it was shown that the single-scattering phase function could be constructed from a linear combination of that for forward and backward paths. We assume that the higher order scattering also can be so approximated.

The scattering phase function provided by double scattered radiation is now obtained from Eq. (16) by its integration over attenuated incoming and outgoing radiation [14]. That is, the scattering phase function in the backward (1) or forward (-1) directions, normalized by division by $\omega^2/(4\pi)$ and denoted Φ_{2s} is then obtained from:

$$\Phi_{2s}(1,\tau) = \int_0^1 2\mu\,\mathrm{d}\mu \int_0^{2\mu} A_0^2(h)B^*(h,\mu)\,\mathrm{d}h, \tag{17}$$

$$\Phi_{2s}(-1,\tau) = \int_0^1 2\mu\,\mathrm{d}\mu \int_0^{2\mu} A_0(h)A_0(2\mu-h)B^*(h,\mu)\,\mathrm{d}h. \tag{18}$$

These integrals are readily determined by substituting into them the definitions of A_0 and B^* from Eqs. (12) and (16), i.e.,

$$\Phi_{2s}(1,\tau) = \frac{1}{3} - T(2\tau) + \frac{2}{3}T(3\tau), \tag{19}$$

$$\Phi_{2s}(-1,\tau) = \frac{4}{3}\Phi_{1s}(-1,\tau) + T(2\tau) + \frac{1}{9}T(4\tau) - \frac{10}{9}T(\tau), \tag{20}$$

where

$$T(\tau) = 0.5\tau^{-2}[1-(1+2\tau)\exp(-2\tau)], \tag{21}$$

is the direct beam transmission, Eq. (7) of [12], and

$$\Phi_{1s}(-1,\tau) = \tau^{-2}[1-(1+2\tau+2\tau^2)\exp(-2\tau)] \tag{22}$$

is the normalized single scatter term, i.e., Eq. (8b) of [12] for forward scatter.

If the terms of Eqs. (19) and (20) are expanded in τ, the resulting terms cancel until $\mathrm{O}(\tau^2)$ where they sum to $2\tau^2$.

The fraction of radiation that is attenuated after scattering is shown in [16] to converge to a constant value independent of scattering order, and assumed here to be adequately estimated at the second order scattering. To approximate this limit, we introduce notation for the directionally averaged first and second order scattering phase functions, i.e., use the bracket symbols $\langle\rangle$ to denote average over direction of radiation: $\langle\Phi_{1s}\rangle = 0.5[\Phi_{1s}(1,\tau) + \Phi_{1s}(-1,\tau)]$, and $\langle\Phi_{2s}\rangle = 0.5[\Phi_{2s}(1,\tau) + \Phi_{2s}(-1,\tau)]$. The probability of an absorption after two scatterings p_a is obtained as 1 minus the second order scattering divided by 1 minus that escaping in up to one scattering, i.e.,

$$p_a = 1 - \langle \Phi_{2s} \rangle / [1 - T(\tau) - \omega \langle \Phi_{1s} \rangle] \tag{23}$$

From Eqs. (19) and (20), this term is seen to approach 7/9 at large τ. At convergence to a constant attenuation probability, the scattered radiation that escapes is isotropic, i.e., follows from p_a independent of outgoing direction. With these assumptions, the series of higher order scatterings can be summed [16] so that third and higher order scatterings are added to the second order scattering to estimate the multi-order scattering as

$$\Phi_{ms}(1, \tau) = \Phi_{2s}(1, \tau) + \omega p_a \langle \Phi_{2s} \rangle / (1 - \omega p_a), \tag{24}$$

$$\Phi_{ms}(-1, \tau) = \Phi_{2s}(-1, \tau) + \omega p_a \langle \Phi_{2s} \rangle / (1 - \omega p_a). \tag{25}$$

The factor $\langle \Phi_{2s} \rangle$ summarizes the product of the escape probability after a second scattering and the absorption that has occurred during the first two orders of scattering. In the limit of perfect scattering, $\omega = 1$, Eqs. (24) and (25) combined with single scattering reduce to a total scattering phase function of

$$\Phi(\mu, \tau) \to 1 - T(\tau) + \Phi_{1s}(\mu, \tau) + \Phi_{2s}(\mu, \tau) - \langle \Phi_{1s} \rangle - \langle \Phi_{2s} \rangle. \tag{26}$$

Fig. 2 shows the multi-order scattering phase function provided by Eqs. (24) and (25). It indicates in particular that: (a) higher order scattering is negligible out to $\tau \approx 0.5$; (b) scattering is independent of direction out to

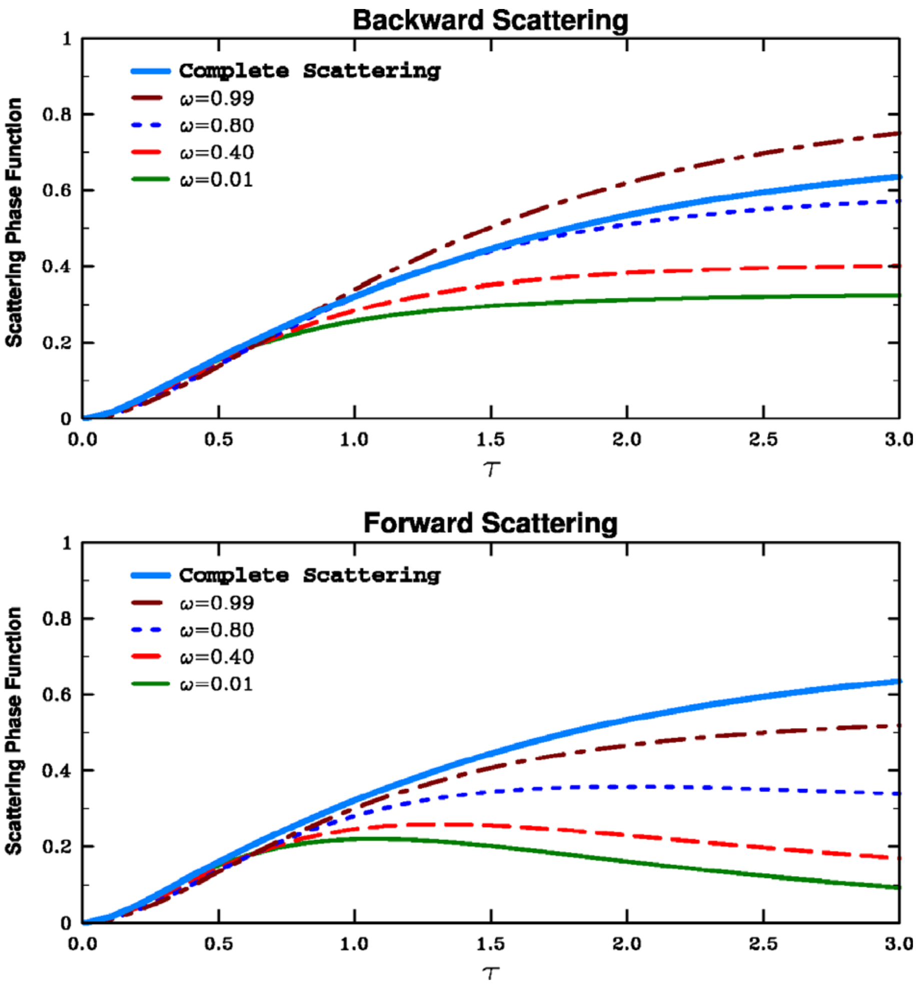

Fig. 2. The analytic normalized scattering phase function for multiple scattering from a sphere of scatters, i.e., not including the first order scattering, and to be multiplied by $\omega^2/(4\pi)$ to remove the normalization. The green solid line approximately corresponds to second order scattering, i.e., $\omega = 0$. Higher order scattering is assumed independent of direction and estimated by construction of an attenuation probability from the second order scattering. The solid blue line shows another estimate of the multiple scattering at $\omega = 1$ to be compared with the average of the forward and backward scattering at $\omega = 0.99$.

$\tau \approx 1$, but backward is twice or more as large as forward scattering by $\tau \approx 2$. the line labeled "complete scattering" shows another approximate expression, Eq. (32), for the average multi-scattering at $\omega = 1$. It agrees with the average of the brown lines at both ends, but is smaller between τ of 1 and 2 by several percent. The relative differences between second order forward and backward scattering are smaller than for first order. However, the neglect of directionality in the higher order scattering suggests the estimate of the directional component of scattering (backward minus forward) may be biased low by up to several percent and perhaps more at the high end of the plot.

4. Obtaining a resolvent kernel for the spherical bush

This section shows how the formal expressions of Section 2 can, in principal, be solved for the scattering to all orders by solution of the scalar integral equation:

$$A(h,\mu) = A_0(h,\mu) + \omega \int_0^{2\mu} B(h,h')A(h,\mu)\,\mathrm{d}h'. \tag{27}$$

Unfortunately, the exact solutions are apparently too complex to provide the simple algorithms needed for climate models. However, they suggest some further approximate expressions appropriate to other regions of parameter space than that given in Eqs. (24)–(26). Although the $B(h,h')$ could be, in principle, any along path representation of the **I** of Eq. (9), we limit the analysis here to the simplest and most approximate version as given by Eq. (14).The inversion of Eq. (27) to solve for A is, in principle, simple because B is recognized as the Green's function for the differential operator **D** where

$$\mathbf{D} = [1 - (0.5/\tau)^2 \mathrm{d}^2/\mathrm{d}h^2], \tag{28}$$

With some rearrangement, Eq. (27) can be written as

$$\mathbf{D}^*(A - A_0) = \omega A_{0,} \tag{29}$$

where

$$\mathbf{D}^* = [1 - \omega - (0.5/\tau)^2 \mathrm{d}^2/\mathrm{d}h^2], \tag{30}$$

provides the eigen-functions of the bush integral equation, i.e.,

$$\Gamma(\pm h) = \exp[\pm 2(1-\omega)^{1/2} h\tau]. \tag{31}$$

The integral equation is inverted by constructing a combination of these eigen-functions that satisfy the condition of outgoing fluxes at the boundaries. Only in the large τ limit are the details simple enough to be integrated over the sphere and so are not provided here since they can also be constructed from the more familiar "two-stream" model solutions (e.g. [19]). The physical interpretation of the integral equation formulation is that incident radiation is attenuated and transferred from the location of initial attenuation to another location where it undergoes its last scattering and exits. The transfer occurs with many scatters whose net attenuation is described by exponential decay with the eigen-function structure. The exiting forward scattered radiation is dominated by these eigen-functions but that in backward direction contains a strong dependence on the entering and final exiting path, i.e., as provided by a modified version of the single-scattering result. A simple result is possible in the limit of $(1-\omega)^{1/2}\tau$ small. In this limit, the transfer along the eigen-function is with negligible attenuation so that significant attenuation only occurs for the entering and exiting photons. Consequently,

$$\Phi_{ms}(\mu,\tau) \approx (1 - 2T(\tau) + T(2\tau)) - \langle \Phi_{1s} \rangle \tag{32}$$

which satisfies $\Phi_{ms}(\mu,\tau) \to 2\tau^2$ as $\tau \to 0$ and $\to 1$, $\tau \to \infty$. The second factor corrects for the loss to single scattering. We have not included a backward–forward asymmetric contribution that will become increasingly significant with large T. Fig. 2 compares this term with the forward and backward multiple scattering as a function of τ for the two approximate expressions Eqs. (24,25). At small values of ω, all these plots are approximated by Eqs. (19) and (20) for second order scattering.

The complete backward and forward scattering phase functions $\Psi(1,\tau)$ *and* $\Psi(-1,\tau)$ are constructed as

$$\Psi(1,\tau) = (\omega/4\pi)[\Phi_{1s}(1,\tau) + \omega\Phi_{ms}(1,\tau)], \tag{33}$$

$$\Psi(-1,\tau) = (\omega/4\pi)[\Phi_{1s}(-1,\tau) + \omega\Phi_{ms}(-1,\tau)], \tag{34}$$

where $\Phi_{ms}(1,\tau)$ is constructed from Eq. (24) or (32), $\Phi_{ms}(-1,\tau)$ from Eq. (25) or (32), $\Phi_{1s}(-1,\tau)$ from Eq. (22) and $\Phi_{1s}(1,\tau)$ is derived in [12] as

$$\Phi_{1s}(1,\tau) = 0.5[1 - T(2\tau)]. \tag{35}$$

The scattering for any angle, $\theta = \cos^{-1}\eta$, between incoming and outgoing radiation is obtained from [12] as

$$\Psi(\eta,\tau) = 0.5[(1+\eta)\Psi(1,\tau) + (1-\eta)\Psi(-1,\tau)]. \tag{36}$$

5. Discussion

Higher order scattering by a spherical bush involves 3D exchange integrals. With a Laplace transform representation, these integrations can be approximated by 1D integrals over exponentials along the paths of entering and exiting radiation. These integrations depend on the location in the sphere where the radiation enters. For simple enough dependences on this location, the integrations over the sphere can be done exactly. This approach yields results for second order scattering in terms of simple analytic expressions.

The analytic expressions for second order scattering are used to develop an approximate estimate for the higher order scattering. Although doubtful in detail for near perfect scattering, it approaches the correct limit of conservative scattering. Solutions to all orders of scattering can be constructed from eigen-functions of the scattering integral equation. The detailed expressions we found were too complicated for the intended use in climate models. However, this approach is useful for development of more approximate expressions in the limit of near perfect scattering. One simple such expression is presented, Eq. (32), that is qualitatively similar, but differs in detail from that obtained as Eq. (26) from the second order scattering approach. Such conservative scattering is of more interest for clouds than bushes, which also involve strongly anisotropic scattering, and so was not pursued further here.

Eq. (36) is simply integrated over view angle to provide albedo of an isolated spherical bush. From the previous quantification of the single-scattering error in [12], the numerical estimation errors are believed to be smaller than those introduced by physical modeling assumptions. The assumption of a sphere can be modified in various ways. All the results derived here can be mapped to a spheroid. The scattering over paths through a sphere are integrable because the vertically projected area for a given distance through the sphere is distributed as a polynomial $P(\mu)$ in the scaling distance μ, and the arguments of exponentials depend linearly on this scaling distance. Any other geometric shape can be similarly characterized, e.g. a cone has the same distribution as a sphere except weighted toward shortest rather than longest path For complex geometries, the fitting could be done by using Monte-Carlo sampling to determine a histogram of optical paths through the object and fitting a low order polynomial to the inferred distribution of paths. Such analyses only provides scattering along the forward and backward paths, and how these should be averaged to estimate scattering in other directions will depend on geometric details and needs to be established for each case. The formalism of Sections 2 can be used for guidance in any such example. More complicated dependences on the scaling distance, e.g. polynomials in the denominators, do not reduce to anything simpler than exponential integrals, e.g. as illustrated by the example in [10].

6. Conclusions

The purpose of this analysis has been to develop a simple approach for determining the reflection of solar radiation from a surface that includes 3D canopy objects. These objects are referred to as bushes and located within the bush is a homogeneous distribution of individual elements scattering isotropically. This approach is intended to provide reflected radiation both for a climate model and as seen by remote sensing instruments so it is necessary to derive the parameters needed by a climate model from the "optical properties" of the bush. These properties can be combined with the reflectance of the underlying soil and reflections from neighboring bushes to fully characterize the partitioning of solar radiative heating between the different such components and the back-reflection to the atmosphere, i.e., the albedo. Analytic solutions are developed in this paper for a sphere of scattering whose optical properties depend only on the optical depth along a radius, the single-scattering albedo of the leaves, and the angle between incoming and outgoing radiation, i.e., sun and view angles. These solutions can be combined with parameterizations for exchanges of radiation between neighboring bushes and underlying soil to construct a 3D description of the land surface radiative properties.

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